Mental Math
Mental Computation
Grade 6

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Introduction

Definitions

It is important to clarify the definitions used around mental math. Mental math in Nova Scotia refers to the entire program of mental math and estimation across all strands. It is important to incorporate some aspect of mental math into your mathematics planning everyday, although the time spent each day may vary. While the *Time to Learn* document requires 5 minutes per day, there will be days, especially when introducing strategies, when more time will be needed. Other times, such as when reinforcing a strategy, it may not take 5 minutes to do the practice exercises and discuss the strategies and answers.

While there are many aspects to mental math, this booklet, *Mental Computation*, deals with fact learning, mental calculations, and computational estimation — mental math found in General Curriculum Outcome (GCO) B. Therefore, teachers must also remember to incorporate mental math strategies from the six other GCOs into their yearly plans for Mental Math, for example, measurement estimation, quantity estimation, patterns and spatial sense. For more information on these and other strategies see *Elementary and Middle School Mathematics: Teaching Developmentally* by John A. Van de Walle.

For the purpose of this booklet, fact learning will refer to the acquisition of the 100 number facts relating the single digits 0 to 9 for each of the four operations. When students know these facts, they can quickly retrieve them from memory (usually in 3 seconds or less). Ideally, through practice over time, students will achieve automaticity; that is, they will abandon the use of strategies and give instant recall. Computational estimation refers to using strategies to get approximate answers by doing calculations in one's head, while mental calculations refer to using strategies to get exact answers by doing all the calculations in one's head.

While we have defined each term separately, this does not suggest that the three terms are totally separable. Initially, students develop and use strategies to get quick recall of the facts. These strategies and the facts themselves are the foundations for the development of other mental calculation strategies. When the facts are automatic, students are no longer employing strategies to retrieve them from memory. In turn, the facts and mental calculation strategies are the foundations for estimation. Attempts at computational estimation are often thwarted by the lack of knowledge of the related facts and mental calculation strategies.

Rationale

In modern society, the development of mental computation skills needs to be a major goal of any mathematical program for two major reasons. First of all, in their day-to-day activities, most people's calculation needs can be met by having well developed mental computational processes. Secondly, while technology has replaced paper-and-pencil as the major tool for complex computations, people need to have well developed mental strategies to be alert to the reasonableness of answers generated by technology.

Besides being the foundation of the development of number and operation sense, fact learning itself is critical to the overall development of mathematics. Mathematics is about patterns and relationships and many of these patterns and relationships are numerical. Without a command of the basic relationships among numbers (facts), it is very difficult to detect these patterns and relationships. As well, nothing empowers students with confidence and flexibility of thinking more than a command of the number facts.

It is important to establish a rational for mental math. While it is true that many computations that require exact answers are now done on calculators, it is important that students have the necessary skills to judge the reasonableness of those answers. This is also true for computations students will do using pencil-and-paper strategies. Furthermore, many computations in their daily lives will not require exact answers. (e.g., If three pens each cost $1.90, can I buy them if I have $5.00?) Students will also encounter computations in their daily lives for which they can get exact answers quickly in their heads. (e.g., What is the cost of three pens that each cost $3.00?)

The Implementation of Mental Computational Strategies

General Approach

In general, a strategy should be introduced in isolation from other strategies, a variety of different reinforcement activities should be provided until it is mastered, the strategy should be assessed in a variety of ways, and then it should be combined with other previously learned strategies.

Introducing a Strategy

The approach to highlighting a mental computational strategy is to give the students an example of a computation for which the strategy would be useful to see if any of the students already can apply the strategy. If so, the student(s) can explain the strategy to the class with your help. If not, you could share the strategy yourself. The explanation of a strategy should include anything that will help students see the pattern and logic of the strategy, be that concrete materials, visuals, and/or contexts. The introduction should also include explicit modeling of the mental processes used to carry out the strategy, and explicit discussion of the situations for which the strategy is most appropriate and efficient. The logic of the strategy should be well understood before it is reinforced. (Often it would also be appropriate to show when the strategy would not be appropriate as well as when it would be appropriate.)

Reinforcement

Each strategy for building mental computational skills should be practised in isolation until students can give correct solutions in a reasonable time frame. Students must understand the logic of the strategy, recognize when it is appropriate, and explain the strategy. The amount of time spent on each strategy should be determined by the students' abilities and previous experiences.

The reinforcement activities for a strategy should be varied in type and should focus as much on the discussion of how students obtained their answers as on the answers themselves. The reinforcement activities should be structured to insure maximum participation. Time frames should be generous at first and be narrowed as students internalize the strategy. Student participation should be monitored and their progress assessed in a variety of ways to help determine how long should be spent on a strategy.

After you are confident that most of the students have internalized the strategy, you need to help them integrate it with other strategies they have developed. You can do this by providing activities that includes a mix of number expressions, for which this strategy and others would apply. You should have the students complete the activities and discuss the strategy/strategies that could be used; or you should have students match the number expressions included in the activity to a list of strategies, and discuss the attributes of the number expressions that prompted them to make the matches.

Assessment

Your assessments of mental math and estimation strategies should take a variety of forms. In addition to the traditional quizzes that involve students recording answers to questions that you give one-at-a-time in a certain time frame, you should also record any observations you make during the reinforcements, ask the students for oral responses and explanations, and have them explain strategies in writing. Individual interviews can provide you with many insights into a student's thinking, especially in situations where pencil-and-paper responses are weak.

Assessments, regardless of their form, should shed light on students' abilities to compute efficiently and accurately, to select appropriate strategies, and to explain their thinking.

Response Time

Response time is an effective way for teachers to see if students can use the mental math and estimation strategies efficiently and to determine if students have automaticity of their facts.

For the facts, your goal is to get a response in 3-seconds or less. You would give students more time than this in the initial strategy reinforcement activities, and reduce the time as the students become more proficient applying the strategy until the 3-second goal is reached. In subsequent grades when the facts are extended to 10s, 100s and 1000s, a 3-second response should also be the expectation.

In early grades, the 3-second response goal is a guideline for the teacher and does not need to be shared with the students if it will cause undue anxiety.

With other mental computational strategies, you should allow 5 to 10 seconds, depending upon the complexity of the mental activity required. Again, in the initial application of the strategies, you would allow as much time as needed to insure success, and gradually decrease the wait time until students attain solutions in a reasonable time frame.

A. Addition — Fact Learning

It is important to note here that by grade 6, students should have mastered their addition, subtraction, multiplication and division number facts. If there are students who have not done this, it is necessary to go back to where the fact learning strategies are introduced and to work from those students are comfortable.

See grade 5 for the grade 6 review.

B. Addition and Subtraction — Mental Calculation

Quick Addition and Subtraction – No Regrouping:

This pencil-and-paper strategy is used when there are more than two combinations in the calculations, but no regrouping is needed and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end. It is important to present these addition questions both horizontally and vertically.

Examples

For example: For 2.327 + 1.441, simply record, starting at the front end, 3.768.

For example: For 7.593 – 2.381, simply record, starting at the front end, 5.212.

Some examples of practice items

Some practice items for numbers in the thousandths are:

7.406 + 2.592 =	4.234 + 2.755 =	12.295 + 7.703
6.234 + 2.604 =	32.107 + 10.882 =	100.236 + 300.543 =
8.947 – 2.231 =	7.076 – 3.055 =	12.479 – 1.236 =
0.735 – 0.214 =	96.983 – 12.281 =	125.443 – 25.123 =

C. Multiplication and Division — Mental Calculation

Some of the following material is review from grade 5, but it is necessary to include it here to consolidate the understanding of multiplying by tenths, hundredths and thousandths; the related division by tens, hundreds and thousands; the reverse of multiplying by tens, hundreds and thousands; the related division by tenths, hundredths and thousandths.

Quick Multiplication – No Regrouping

Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all the combinations in their heads starting at the front end.

Examples

For example: For 52 × 3, simply record, starting at the front end, 150 + 6 = 156.

For example: Foe 423 × 2, simply record, starting at the front end, 800 + 40 + 6 = 846.

Some examples of practice items

Here are some practice items.

43 × 2 =	72 × 3=	84 × 2 =
142 × 2 =	803 × 3 =	342 × 2 =
12.3 × 3 =	14 3 × 2 =	63 000 × 2 =
1 220 × 3 =	42 000 × 4 =	43.42 × 2 =

Quick Division – No Regrouping

Note: This pencil-and-paper strategy is used when there is no regrouping and the questions are presented visually instead of orally. It is included here as a mental math strategy because students will do all of the combinations in their heads starting at the front end.

Examples

For 640 ÷ 2, simply record, starting at the front end, 300 + 20 = 320.

For 1 290 ÷ 3, simply record, starting at the front end, 400 + 30 = 430.

Some examples of practice items

Here are some practice items.

360 ÷ 3 =	420 ÷ 2 =	707 ÷ 7 =
105 ÷ 5 =	426 ÷ 6 =	320 ÷ 8 =
490 ÷ 7 =	505 ÷ 5 =	819 ÷ 9 =
328 ÷ 4 =	103 ÷2 =	455 ÷ 5 =
2 107 ÷ 7 =	7 280 ÷ 8 =	3 570 ÷ 7 =
3 612 ÷ 6 =	248 000 ÷ 8 =	279 000 ÷ 9 =

Multiplying & Dividing by 10, 100 and 1000

Multiplication: This strategy involves keeping track of how the place values have changed.

Multiplying by 10 increases all the place values of a number by one place . For 10 × 67, think: the 6 tens will increase to 6 hundreds and the 7 ones will increase to 7 tens; therefore, the answer is 670.

Multiplying by 100 increases all the place values of a number by two places. For 100 × 86, think: the 8 tens will increase to 8 thousands and the 6 ones will increase to 6 hundreds; therefore, the answer is 8 600. It is necessary that students use the correct language when orally answering questions where they multiply by 100. For example, the answer to 100 × 86 should be read as 86 hundred and not 8 thousand 6 hundred.

Multiplying by1000 increases all the place values of a number by three places. For 1000 × 45, think: the 4 tens will increase to 40 thousands and the 5 ones will increase to 5 thousands; therefore, the answer is 45 000. It is necessary that students use the correct language when orally answering questions where they multiply by 1000. For example the answer to 1000 × 45 should be read as 45 thousand and not 4 ten thousands and 5 thousand.

Some examples of practice items

Some mixed practice items are:

10 × 53 =	10 × 34 =	87 × 10 =
10 × 20 =	47 × 10 =	78 × 10 =
92 × 10 =	10 × 66 =	40 × 10 =
100 × 7 =	100 × 2 =	100 × 15 =
100 × 74 =	100 × 39 =	37 × 100 =
10 × 10 =	55 × 100 =	100 × 83 =
100 × 70 =	90 × 90 =	40 × 100 =
1 000 × 6 =	1 000 × 14 =	83 × 1 000 =
$73 × 1 000 =	$20 × 1 000 =	16 × $1 000 =
5m = ___ cm	8m = ___cm	3m =___cm
$3 × 10 =	$7 × 10 =	$50 × 10 =
3 m = _____mm	7m = ____	4.2m = ___m
6.2m = ____mm	6cm = ___mm	9km = ____m
7.7km = ____m	3dm = ____mm	3dm = ____ cm
10 × 3.3 =	4.5 × 10 =	0.7 × 10 =
8.3 × 10 =	7.2 × 10 =	10 × 4.9 =
100 × 2.2 =	100 × 8.3 =	100 × 9.9 =
7.54 × 10 =	8.36 x10 =	10 × 0.3 =
100 × 0.12 =	100 × 0.41 =	100 × 0.07 =
3.78 × 100 =	1 000 × 2.2 =	1 000 × 43.8 =
1 000 × 5.66 =	8.02 × 1 000 =	0.04 × 1 000 =

Dividing by tenths (0.1), hundredths (0.01) and thousandths (0.001)

When students fully understand decimal tenths and hundredths, they will be able to use this knowledge in understanding multiplication and division by tenths and hundredths in mental math situations.

Multiplying by 10s, 100s and 1 000s, is similar to dividing by tenths, hundredths and thousandths.

1) Dividing by tenths increases all the place values of a number by one place.
2) Dividing by hundredths increases all the place values of a number by two places.

Examples

1) For 3 ÷ 0.1, think: the 3 ones will increase to 3 tens, therefore the answer is 30.

   For 0.4 ÷ 0.1, think: the 4 tenths will increase to 4 ones, therefore the answer is 4.

2) For 3 × 0.01, think: the 3 ones will increase to 3 hundreds, therefore the answer is 300.

   For 0.4 ÷ 0.01, think: the 4 tenths will increase to 4 tens, therefore the answer is 40.

   For 3.7 ÷ 0.01, think: the 3 ones will increase to 3 hundreds and the 7 tenths will increase to 7 tens, therefore, the answer is 37.

Some examples of practice items

1) Here are some practice items:

5 ÷ 0.1 =	7 ÷ 0.1 =	23 ÷ 0.1 =
46 ÷ 0.1 =	0.1 ÷ 0.1 =	2.2 ÷ 0.1 =
0.5 ÷ 0.1 =	1.8 ÷ 0.1 =	425 ÷ 0.1 =
0.02 ÷ 0.1 =	0.06 ÷ 0.1 =	0.15 ÷ 0.1 =
14.5 ÷ 0.1 =	1.09 ÷ 0.1 =	253.1 ÷ 0.1 =

2) Here are some practice items:

4 ÷ 0.01 =	7 ÷ 0.01 =	4 ÷ 0.01 =
1 ÷ 0.01 =	9 ÷ 0.01 =	0.5 ÷ 0.01 =
0.2 ÷ 0.01 =	0.3 ÷ 0.01 =	0.1 ÷ 0.01 =
0.8 ÷ 0.01 =	5.2 ÷ 0.01 =	6.5 ÷ 0.01 =
8.2 ÷ 0.01 =	9.7 ÷ 0.01 =	*17.5 ÷ 0.01 =

Dividing by Ten, Hundred and Thousand

Division: This strategy involves keeping track of how the place values have changed.

Dividing by10 decreases all the place values of a number by one place.

Dividing by 100 decreases all the place values of a number by two places.

Dividing by 1000 decreases all the place values of a number by three places.

Examples

For 340 ÷ 10, think: the 3 hundreds will decrease to 3 tens and the 4 tens will decrease to 4 ones; therefore, the answer is 34.

For, 7 500 ÷ 100; think: the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75.

For, 63 000 ÷ 1000; think: the 6 ten thousands will decrease to 6 tens and the 3 thousands will decrease to 3 ones; therefore, the answer is 63.

Some examples of practice items

Here are some mixed practice items:

80 ÷ 10 =	60 ÷ 10 =	100 ÷ 10 =
420 ÷ 10 =	790 ÷ 10 =	360 ÷ 10 =
1 200 ÷ 10 =	4 400 ÷ 10 =	900 ÷ 100 =
700 ÷ 100 =	3 000 ÷ 100 =	4 000 ÷ 100 =
2 400 ÷ 100 =	3 800 ÷ 100 =	37 000 ÷ 100 =
7 000 ÷ 1 000 =	34 000 ÷ 1 000 =	29 000 ÷ 1 000 =
80 000 ÷ 1 000 =	96 000 ÷ 1 000 =	100 000 ÷ 1 000 =
2 000 ÷ 1 000 =	13 000 ÷ 1 000 =	750 000 ÷ 1 000 =

Division when the divisor is a multiple of 10 and the dividend is a multiple of the divisor.

Division by a power of ten should be understood to result in a uniform "shrinking" of hundreds, tens and units which could be demonstrated and visualized with base -10 blocks.

Example

For 400 ÷ 20, think: 400 shrinks to 40 and 40 divided by 2 is 20.

Some examples of practice items

Here are some practice items:

500 ÷ 10=	700 ÷ 10 =	900 ÷ 10 =
900 ÷ 30 =	600 ÷ 20 =	4 000 ÷ 10 =
8 000 ÷ 40 =	120 ÷ 10 =	240 ÷ 40 =
12 000 ÷ 20 =	2 000 ÷ 50 =	18 000 ÷ 600 =

Division using the Think Multiplication strategy.

This is a convenient strategy to use when dividing mentally. For example, when dividing 60 by 12, think: "What times 12 is 60?"

This could be used in combination with other strategies.

Example

For 920 ÷ 40, think: "20 groups of 40 would be 800, leaving 120, which is 3 more groups of 40 for a total of 23 groups.

Some examples of practice items

Some practice items are:

240 ÷ 12 =	3 600 ÷ 12 =	660 ÷ 30 =
880 ÷ 40 =	1 260 ÷ 60 =	690 ÷ 30 =
1 470 ÷ 70 =	6 000 ÷ 12 =	650 ÷ 50 =

Multiplication and Division of tenths, hundredths and thousandths.

Multiplying by tenths, hundredths and thousandths is similar to dividing by tens, hundreds and thousands.

This strategy involves keeping track of how the place values have changed.

1) Multiplying by 0.1 decreases all the place values of a number by one place.
2) Multiplying by 0.01 decreases all the place values of a number by two places.
3) Dividing by 100 decreases all the place values of a number by two places.
4) Multiplying by 0.001 decreases all the place values of a number by three places.
5) Dividing by 1000 decreases all the place values of a number by three places .

Example

1) For 5 × 0.1, think: the 5 ones will decrease to 5 tenths; therefore, the answer is 0.5.

   For, 0.4 × 0.1, Think: the 4 tenths will decrease to 4 hundredths, therefore the answer is 0.04.

2) For 5 × 0.01, think: the 5 ones will decrease to 5 hundredths, therefore the answer is 0.05.

   For, 0.4 × 0.01, think: the 4 tenths will decrease to 4 thousandths, therefore the answer is 0.004.

3) For, 7 500 ÷ 100; think: the 7 thousands will decrease to 7 tens and the 5 hundreds will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one hundredth and dividing by 100.

4) For 5 × 0.001, think: the 5 ones will decrease to 5 thousandths; therefore, the answer is 0.005.

   For, 8 × 0.001, think: the 8 ones will decrease to 8 thousandths; therefore, the answer is 0.008.

5) For, 75 000 ÷ 1000; think: the 7 ten thousands will decrease to 7 tens and the 5 thousands will decrease to 5 ones; therefore, the answer is 75. This is an opportunity to show the relationship between multiplying by one thousandth and dividing by 1000.

Some examples of practice items

1) Tenths

6 × 0.1 =	8 × 0.1 =	3 × 0.1 =
9 × 0.1 =	1 × 0.1 =	12 × 0.1 =
72 × 0.1 =	136 × 0.1 =	406 × 0.1 =
0.7 × 0.1 =	0.5 × 0.1 =	0.1 × 10 =
1.6 × 0.1 =	0.1 × 84 =	0.1 × 3.2 =

2) Hundredths:

6 × 0.01 =	8 × 0.01 =	1.2 × 0.01 =
0.5 × 0.01 =	0.4 × 0.01 =	0.7 × 0.01 =
2.3 × 0.01 =	3.9 × 0.01 =	10 × 0.01 =
100 × 0.01 =	330 × 0.01 =	46 × 0.01 =

3) Here are some practice items:

400 ÷ 100 =	900 ÷ 100 =	6 000 ÷ 100 =
4 200 ÷ 100 =	7 600 ÷ 100 =	8 500 ÷ 100 =
9 700 ÷ 100 =	4 400 ÷ 100 =	10 000 ÷ 100 =
600 pennies = $_____	1 800 pennies =$_____	56 000 pennies = $____

4) Her are some practice items:

3 × 0.001 =	7 × 0.001 =	80 × 0.001 =
21 × 0.001 =	45 × 0.001 =	12 × 0.001 =
600 × 0.001 =	325 × 0.001 =	4 261 × 0.001 =
4mm = ____m	9mm = _____m	6m = ____km

5) Here are some practice items:

82 000 ÷ 1 000 =	98 000 ÷ 1 000 =	12 000 ÷ 1 000 =
66 000 ÷ 1 000 =	70 000 ÷ 1 000 =	100 000 ÷ 1 000 =
430 000 ÷ 1 000 =	104 000 ÷ 1 000 =	4 500 ÷ 1 000 =
77 000m = _____km	84 000m = ____km	7 700m = _____km

Compensation

This strategy for multiplication involves changing one of the factors to a ten, hundred or thousand; carrying out the multiplication; and then adjusting the answer to compensate for the change that was made. This strategy could be carried out when one of the factors is near ten, hundred or thousand.

Examples

For 6 × $4.98, think: 6 times 5 dollars less 6 × 2 cents, therefore $30 subtract $0.12 which is $29.88. The same strategy applies to decimals. This strategy works well with 8s and 9s.

For example: For 3.99 × 4, think: 4 × 4 is 16 subtract 4 × 0.01(0.04) which is 15.96.

Some examples of practice items

Here are some practice items:

4.98 × 2 =	5.99 × 7 =	$6.98 × 3 =
$9.99 × 8 =	$19.99 × 3 =	$49.98 × 4 =
6.99 × 9 =	20.98 × 2 =	$99.98 × 5 =

Halving and Doubling

This strategy involves halving one factor and doubling the other factor in order to get two new factors that are easier to calculate. While the factors have changed, the product is equivalent, because multiplying by one-half and then by 2 is equivalent to multiplying by 1, which is the multiplicative identity. Halving and doubling is a situation where students may need to record some sub-steps.

Examples

For example: For 42 × 50, think: one-half of 42 is 21 and 50 doubled is 100; therefore, 21 × 100 is 2 100.

For example: For 500 × 88, think: double 500 to get 1000 and one-half of 88 is 44; therefore, 1 000 × 44 is 44 000.

For example: For 12 × 2.5, think: one-half of 12 is 6 and double 2.5 is 5; therefore, 6 × 5 is 30.

For example: For 4.5 × 2.2, think: double 4.5 to get 9 and one-half of 2.2 is 1.1; therefore, 9 × 1.1 is 9.9.

For example: For 140 × 35, think: one-half of 140 is 70 and double 35 is 70; therefore 70 × 70 is 4 900.

Some examples of practice items

Here are some practice items:

86 × 50 =	50 × 28 =	64 × 500 =
500 × 46 =	52 × 50 =	500 × 70 =
18 × 2.5 =	2.5 × 22 =	86 × 2.5 =
0.5 × 120 =	3.5 × 2.2 =	1.5 × 6.6 =
180 × 45 =	160 × 35 =	140 × 15 =

Front End Multiplication or the Distributive Principle in 10s, 100s and 1000s

Note: This strategy involves finding the product of the single-digit factor and the digit in the highest place value of the second number, and adding to this product a second sub-product. This strategy is also known as the distributive principle.

Examples

1) For, 62 × 3, think: 3 times 6 tens is 18 tens, or 180; and 3 times 2 is 6; so 180 plus 6 is 186.
2) For, 2 × 706, think: 2 times 7 hundreds is 14 hundreds, or 1 400; and 2 times 6 is 12; so 1 400 plus 12 is 1412.
3) For, 5 × 6 100, think: 5 times 6 thousands is 30 thousands, or 30 000; and 5 times 100 is 500; so 30 000 plus 500 is 30 500.

Some examples of practice items

1) Some practice items in the 10s are:

53 × 3 =	32 × 4 =	41 × 6 =
29 × 2 =	83 × 3 =	75 × 3 =
62 × 4 =	92 × 5 =	35 × 4 =

2) Some practice items in the 100s are:

3 × 503 =	209 × 9 =	703 × 8 =
606 × 6 =	503 × 2 =	8 04 × 6 =
309 × 7 =	122 × 4 =	320 × 3 =
410 × 5 =		

3) Some practice items in the 1 000s are:

3 × 4 200 =	4 × 2 100 =	6 × 3 100 =
5 × 5 100 =	2 × 4 300 =	3 × 3 200 =
2 × 4 300 =	7 × 2 100 =	4 × 4 200 =

Finding Compatible Factors

This strategy for multiplication involves looking for pairs of factors whose product is a power of ten and re-associating the factors to make the overall calculation easier. This is possible because of the associative property of multiplication.

Examples

1) For 25 × 63 × 4, think: 4 times 25 is 100, and 100 times 63 is 6 300.

   For 2 × 78 × 500, think: 2 times 500 is 1 000, and 1 000 times 78 is 78 000.

   For 5 × 450 × 2, think: 2 times 5 is 10, and 10 times 450 is 4 500.

2) Sometimes this strategy involves factoring one of the factors to get a compatible.

   For 25 × 28, think: 28(7 × 4) has 4 as a factor, so 4 times 25 is 100, and 100 times 7 is 700.

   For 68 × 500, think: 68 (34 x2) has 2 as a factor, so 500 times 2 is 1 000, and 1 000 times 34 is 34 000

Some examples of practice items

1) Here are some practice items:

5 × 19 × 2 =	2 × 43 × 50 =	4 × 38 × 25 =
500 × 86 × 2 =	250 × 56 × 4 =	40 × 25 × 33 =
250 × 67 × 4 =	40 × 37 × 25 =	5 000 × 9 × 2 =

2) Here are some practice items:

25 × 32 =	50 × 25 =	12 × 25 =
24 × 500 =	250 × 16 =	500 × 36 =
250 × 8 =	16 × 2 500 =	5 000 × 6 =

Using Division Facts for Tens, Hundreds and Thousands.

This strategy applies to dividends of tens, hundreds and thousands divided by a single digit divisor. There would be only one non-zero digit in the quotient.

Example

60 ÷ 3, think: 6 ÷ 3 is 2 and therefore 60 ÷ 3 is 20.

Some examples of practice items

Tens:

90 ÷ 3 =	60 ÷ 2 =	40 ÷ 5 =
120 ÷ 6 =	210 ÷ 7 =	240 ÷ 6 =
180 ÷ 9 =	450 ÷ 9 =	560 ÷ 8 =

Hundreds:

800 ÷ 4 =	200 ÷ 1 =	600 ÷ 3 =
3 500 ÷ 7 =	1 600 ÷ 4 =	7 200 ÷ 8 =
7 200 ÷ 9 =	2 000 ÷ 4 =	2 400 ÷ 3 =
2 400 ÷ 4 =	2 400 ÷ 8 =	2 400 ÷ 6 =
8 100 ÷ 9 =	4 900 ÷ 7 =	3 000 ÷ 5 =

Thousands:

4 000 ÷ 2 =	3 000 ÷ 1 =	9 000 ÷ 3 =
35 000 ÷ 5 =	72 000 ÷ 9 =	36 000 ÷ 6 =
40 000 ÷ 8 =	12 000 ÷ 4 =	64 000 ÷ 8 =
28 000 ÷ 4 =	42 000 ÷ 6 =	10 000 ÷ 2 =

Partitioning the Dividend

This strategy involves partitioning the dividend into two parts, both of which are easily divided by the given divisor. Students should look for ten, hundred or thousand that is an easy multiple of the divisor and that is close to, but less than, the given dividend.

Examples

For 372 ÷ 6, think: (360 + 12) ÷ 6, so 60 + 2 is 62.

For 3150 ÷ 5, think: (3 000 + 150) ÷ 5, so 600 + 30 is 630.

Some examples of practice items

Here are some practice items:

248 ÷ 4 =	224 ÷ 7 =	504 ÷ 8 =
432 ÷ 6 =	344 ÷ 8 =	1 720 ÷ 4 =
8 280 ÷ 9 =	5 110 ÷ 7 =	3 320 ÷ 4 =

Compensation

This strategy for division involves increasing the dividend to an easy multiple of ten, hundred or thousand to get the quotient for that dividend, and then adjusting the quotient to compensate for the increase.

Example

For 348 ÷ 6, think: 348 is about 360 and 360 ÷ 6 is 60 but that is 12 too much; so each of the 6 groups will need to be reduced by 2, so the quotient is 58.

Some examples of practice items

Here are some practice items:

304 ÷ 8 =	228 ÷ 6 =	476 ÷ 7 =
295 ÷ 5 =	352 ÷ 4 =	264 ÷ 3 =
261 ÷ 9 =	1 393 ÷ 7 =	4 188 ÷ 6 =

Balancing For a Constant Quotient

This strategy involves changing a given division question to an equivalent question that will have the same quotient by multiplying both the divisor and the dividend by the same amount. This is done to make the actual dividing process simpler.

Example

For 125 ÷ 5, think: I could multiply both 5 and 125 by 2 to get 250 ÷ 10, which is easy to do. The quotient is 25.

For 120 ÷ 2.5, think: I could multiply both 2.5 and 120 by 4 to get 480 ÷ 10, which is easy to do. The quotient is 48.

For 23.5 ÷ 0.5, think: I could multiply both 23.5 and 0.5 by 2 to get 47 ÷ 1, so the quotient is obviously 47.

Some examples of practice items

Here are some practice items:

140 ÷ 5 =	120 ÷ 25 =	135 ÷ 0.5 =
110 ÷ 2.5 =	320 ÷ 5 =	1200 ÷ 25 =
32.3 ÷ 0.5 =	40 ÷ 2.5 =	250 ÷ 2.5 =

Addition, Subtraction, Multiplication and Division — Computational Estimation

It is essential that estimation strategies are used by students before attempting pencil/paper or calculator computation to help them find "ball park" or reasonable answers.

When teaching estimation strategies, it is necessary to use the language of estimation with your students. Some of the common words and phrases are: about, just about, between, a little more than, a little less than, close, close to and near.

Rounding:

Examples

1) Here are some examples of rounding multiplication questions with a double digit factor by a triple digit factor.

   To round 688 × 79, think: 688 rounds to 700 and 79 rounds to 80, and 700 times 80 is 56 000.

2) Here are some examples of rounding multiplication questions when there are two of the following kinds of factors, one a 3-digit number with a 5or larger in the tens, and the other a 2-digit number with a 5or greater in the ones. Consider rounding the smaller factor up and the larger factor down to give a more accurate estimate. For example, 653 × 45 done with a conventional rounding rule would be 700 × 50 = 35 000, which would not be close to the actual product of 29 385.Using the rounding strategy above, the 45 would round to 50 and the 653 would round to 600, giving an estimate of 30 000, much closer to the actual product. (When both numbers would normally round up, the above rule does not hold true.)

   To round 763 × 36, round 763 (the larger number, down to 700) and round 36 (the smaller number, up to 40) which equals 700 × 40 = 28 000. This produces a closer estimate than rounding to 800 × 40 = 32 000, when the actual product is 27 468.

3) Some examples of rounding division questions with a double digit divisor and a triple digit dividend are:

   For example, to round 789 ÷ 89, round 89 to 90 and think: "90 multiplied by what number would give an answer close to 800(789 rounded)? Since 9 × 9 = 81, therefore 800 ÷ 90 is about 9.

4) Here are some examples of rounding division questions with a 2-digit divisor where you might convert the question to have a single digit divisor.

   For example, 7 843 ÷ 30, think of it as 750 tens ÷ 3 tens to get 250.

Some examples of practice items

1) Here are some practice items:

593 × 41 =	687 × 52 =	708 × 49 =
879 × 22 =	912 × 11 =	384 × 68 =
295 × 59 =	3 950 × 31 =	7 011 × 49 =

2) Here are some practice items:

365 × 27 =	463 × 48 =	567 × 88 =

88 × 473 =	87 × 371 =	658 × 66 =
562 × 48 =	363 × 82 =	972 × 87 =

3) Here are some practice items:

411 ÷ 19 =	360 ÷ 71 =	461 ÷ 92 =
651 ÷ 79 =	810.3 ÷ 89 =	317 ÷ 81 =
233 ÷ 29 =	501 ÷ 71 =	479.95 ÷ 59 =

4) Here are some practice items:

3 610 ÷ 70 =	6 504 ÷ 80 =	2 689 ÷ 90 =
3 989 ÷ 40 =	2 601 ÷ 50 =	8 220 ÷ 90 =
1 909 ÷ 30 =	3 494 ÷ 60 =	1 7 17 ÷ 20 =

Front End Addition, Subtraction and Multiplication

Note: This strategy involves combining only the values in the highest place value to get a “ball- park” figure. Such estimates are adequate in many circumstances. Although estimating to tenths and hundredths is included here, it is most important to estimate to the nearest whole number.

Estimate

1) To estimate 0. 093 + 4.236, think: 0.1 + 4.2 = 4.3(to the nearest tenth).

   To estimate 0.491 + 0.321, think: 0.4 + 0.3 = 0.7(to nearest tenth).

   To estimate 3.871 + 0.124, think: 3 + 0 = 3 (to nearest whole number).

2) To estimate 5.711 – 3.421, think: 5.7 – 3.4 = 2.3(to nearest tenth).

   To estimate 3.871 – 0.901, think: 4 – 1 = 3(to nearest whole number).

3) To estimate 3 125 × 6, think: 3 000 × 6 is 6 groups of 18 thousands, or 18 000.

   To estimate 42 175 × 4, think: 40 000 × 4 is 4 groups of 4 ten thousands, or 160 000.

4) To estimate 3 × 4.952, think: 4 × 5 or 20.

   To estimate 63.141 × 8, think: 60 × 8 or 480.

   To estimate 5 × 0.897, think: 5 × 1, or 5.

Some examples of practice items

1) Some practice items for estimating addition of decimal numbers to tenths and whole numbers are:

   Estimate to the nearest tenth:

0.312 + 0.222 =	0.435 + 0.178 =	0.701 + 0.001 =
3.416 + 0.495 =	2.104 + 2.706 =	10.673 + 20.241 =
0.013 + 0.615 =	0.914 + 0.231 =	1.841 + 1.314 =
0.716 + 0.031 =	0.442 + 0.234 =	0.012 + 0.003 =
3.130 + 3.203 =	100.004 + 100.123 =	18.001 + 0.131 =
0.001 + 0.002 =	3.146 + 2.736 =	0.112 + 0.004 =

2) Some practice items for estimating subtraction of decimal numbers to tenths and whole numbers are:

0.512 – 0.111 =	3.041 – 0.985 =	5.601 – 4.123 =
0.81 – 0.09 =	15.3 – 10.1 =	4.312 – 0.98 =
7.032 – 0.095 =	0.321 – 0.095 =	12.001 – 9.807 =

3) Some practice items for estimating multiplication of numbers in the 1 000s are:

7 200 × 3 =	8 112 × 9 =	3 009 × 5 =
63 000 × 2 =	71 411 × 6 =	80 × 346 =
52 100 × 7 =	73 111 × 4 =	93 010 × 9 =

4) Some practice items for estimating multiplication of numbers in the thousandths by a single digit whole number are:

5 × 3.171 =	7.952 × 7 =	6 × 12.013 =
78.141 × 2 =	87.956 × 8 =	100.123 × 3 =
98.110 × 4 =	6 × 43.333 =	202.273 × 8 =

Front End Division:

Note: This strategy involves rounding the dividend to a number related to a factor of the divisor and then determining in which place value the first digit of the quotient belongs, to get a "ball- park" answer. Such estimates are adequate in many circumstances.

Example

For 425 ÷ 8, round the 425 to 400, we know that the first digit in the quotient is a 5 (5 × 8 = 40) and it is in the tens place, therefore, the quotient is 50.

Some examples of practice items

Here are some practice items:

191 ÷ 3 =	351 ÷ 4 =	735 ÷ 8 =
411 ÷ 6 =	735 ÷ 9 =	276.5 ÷ 9 =
398.4 ÷ 5 =	182 ÷ 2 =	1701 ÷ 2 =

Adjusted Front End or Front End with Clustering

Examples

1) Here are some practice examples for estimating multiplication of double digit factors by double and triple digit factors. Here students may use paper and pencil to record part of the answer.

To estimate 93 × 41, think: 90 × 40 is 40 groups of 9 tens, or 3 600; and 3 × 40 is 40 groups of 3, or 120; 3 600 plus 120 is 3 720.

To estimate 241 × 29, think: 200 × 30 is 200 groups of 3 tens, or 6 000, and 30 groups of 40, or 1 200, and 6 000 plus 1 200 is 7 200.

2) Some practice examples for estimating multiplication of numbers in tenths and hundredths by double digit numbers are:

To estimate 6.1 × 23.4, think: 6 times 20 (120) plus 6 × 3 (18) is 138 and a little more is 140. Or, think of 23.4 as 25 and 25 × 6 is 150.

Some examples of practice items

1) Here are some practice items:

47 × 22 =	86 × 39 =	58 × 49 =
61 × 79 =	222 × 21 =	584 × 78 =
672 × 58 =	481 × 19 =	352 × 61 =

Here are some practice items:

38.2 × 5.9 = (30 × 6) + (8 × 6) = 180 + 48 = 228 plus a little more = 230.

43.1 × 4.1 =	57.2 × 6.9 =	63.1 × 2.1 =
48.3 × 3.2 =	91.2 × 1.9 =	84.3 × 6.1 =
73.3 × 4.1 =	55.1 × 5.1 =	87.3 × 6.2 =

Doubling for Division

Doubling for division involves rounding and doubling both the dividend and divisor. This, does not change the solution but can produce "friendlier" divisors.

Example

For 2 223 ÷ 5 can be thought of as 4 448 ÷ 10, or about 445. It is important that the students understand why this works. See math guide 5- 42.

For 1 333.97 ÷ 5 can be thought of as 2 668 ÷ 10, or about 266.

Some examples of practice items

Here are some practice items:

243 ÷ 5 =	403 ÷ 5 =	231.95 ÷ 5 =
3 212.11 ÷ 5 =	1 343.97 ÷ 5 =	2 222.89 ÷ 5 =